EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. If $A - 2B = \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}$ and $2A - 3B = \begin{bmatrix} -2 & 5 \\ 0 & 7 \end{bmatrix}$, then matrix B is equal to -

(A) $\begin{bmatrix} -4 & -5 \\ -6 & -7 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 6 \\ -3 & 7 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ (D) $\begin{bmatrix} 6 & -1 \\ 0 & 1 \end{bmatrix}$

2. If $A_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then $A_\alpha A_\beta$ is equal to -

(A) $A_{\alpha+\beta}$ (B) $A_{\alpha\beta}$ (C) $A_{\alpha-\beta}$ (D) none of these

3. If number of elements in a matrix is 60 then how many different order of matrix are possible -

(A) 12 (B) 6 (C) 24 (D) none of these

4. Matrix A has x rows and x + 5 columns. Matrix B has y rows and 11 – y columns. Both AB and BA exist, then -

(A) x = 3, y = 4 (B) x = 4, y = 3 (C) x = 3, y = 8 (D) x = 8, y = 3

5. If $A^2 = A$, then$(I + A)^4$ is equal to -

(A) I + A (B) I + 4A (C) I + 15A (D) none of these

6. If the product of n matrices $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\ldots\ldots\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ is equal to the matrix $\begin{bmatrix} 1 & 378 \\ 0 & 1 \end{bmatrix}$ then the value of n is equal to -

(A) 26 (B) 27 (C) 377 (D) 378

7. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(aI_2 + bA)^2 = A$, then -

(A) $a = b = \sqrt{2}$ (B) $a = b = 1/\sqrt{2}$ (C) $a = b = \sqrt{3}$ (D) $a = b = 1/\sqrt{3}$

8. If A is a skew symmetric matrix such that $A^TA = I$, then A^{4n-1} $(n \in N)$ is equal to -

(A) $-A^T$ (B) I (C) – I (D) A^T

9. If $AA^T = I$ and det(A) = 1, then -

(A) Every element of A is equal to it's co-factor.

(B) Every element of A and it's co-factor are additive inverse of each other.

(C) Every element of A and it's co-factor are multiplicative inverse of each other.

(D) None of these

10. Which of the following is an orthogonal matrix -

(A) $\begin{bmatrix} 6/7 & 2/7 & -3/7 \\ 2/7 & 3/7 & 6/7 \\ 3/7 & -6/7 & 2/7 \end{bmatrix}$ (B) $\begin{bmatrix} 6/7 & 2/7 & 3/7 \\ 2/7 & -3/7 & 6/7 \\ 3/7 & 6/7 & -2/7 \end{bmatrix}$

(C) $\begin{bmatrix} -6/7 & -2/7 & -3/7 \\ 2/7 & 3/7 & 6/7 \\ -3/7 & 6/7 & 2/7 \end{bmatrix}$ (D) $\begin{bmatrix} 6/7 & -2/7 & 3/7 \\ 2/7 & 2/7 & -3/7 \\ -6/7 & 2/7 & 3/7 \end{bmatrix}$

11. If A is an orthogonal matrix & | A | = –1, then A^T is equal to -

(A) –A (B) A (C) –(adj A) (D) (adj A)

12. Let A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and 10B = $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of matrix A, then α is -

(A) –2 (B) –1 (C) 2 (D) 5

13. Let the matrix A and B be defined as A = $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 3 & 1 \\ 7 & 3 \end{bmatrix}$ then the value of Det.$(2A^9B^{-1})$, is -

(A) 2 (B) 1 (C) –1 (D) –2

14. If $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then matrix A equals -

(A) $\begin{bmatrix} 7 & 5 \\ -11 & -8 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 7 & 1 \\ 34 & 5 \end{bmatrix}$ (D) $\begin{bmatrix} 5 & 3 \\ 13 & 8 \end{bmatrix}$

15. If A = $\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$ and $f(x) = 1 + x + x^2 + \ldots\ldots + x^{16}$, then $f(A)$ =

(A) 0 (B) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 5 \\ 1 & 1 \end{bmatrix}$

16. If M = $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $M^2 - \lambda M - I_2 = O$, then λ equals -

(A) –2 (B) 2 (C) –4 (D) 4

17. If A = $\begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix}$, B = $\begin{bmatrix} -1 & 4 \\ 2 & 3 \end{bmatrix}$ and ABC = $\begin{bmatrix} 4 & 8 \\ 3 & 7 \end{bmatrix}$, then C equals -

(A) $\frac{1}{66}\begin{bmatrix} 72 & -32 \\ 57 & -29 \end{bmatrix}$ (B) $\frac{1}{66}\begin{bmatrix} -54 & -110 \\ 3 & 11 \end{bmatrix}$ (C) $\frac{1}{66}\begin{bmatrix} -54 & 110 \\ 3 & -11 \end{bmatrix}$ (D) $\frac{1}{66}\begin{bmatrix} -72 & 32 \\ -57 & 29 \end{bmatrix}$

18. If P is a two-rowed matrix satisfying $P^T = P^{-1}$, then P can be -

(A) $\begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ (B) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ (C) $\begin{bmatrix} -\cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$ (D) none of these

19. If A = $\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then | A | | Adj A | is equal to -

(A) a^{25} (B) a^{27} (C) a^{81} (D) none of these

20. If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, then $19A^{-1}$ is equal to -

(A) A^T (B) 2A (C) $\frac{1}{2}A$ (D) A

<u>SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)</u>

21. If A and B are square matrices of same order, then which of the following is correct -

(A) A + B = B + A (B) A + B = A – B
(C) A – B = B – A (D) AB = BA

22. A square matrix can always be expressed as a

(A) sum of a symmetric matrix and skew symmetric matrix of the same order
(B) difference of a symmetric matrix and skew symmetric matrix of the same order
(C) skew symmetric matrix
(D) symmetric matrix

23. Choose the correct answer :

(A) every scalar matrix is an identity matrix.

(B) every identity matrix is a scalar matrix

(C) transpose of transpose of a matrix gives the matrix itself.

(D) for every square matrix A there exists another matrix B such that AB = I = BA.

24. Let a_{ij} denote the element of the i^{th} row and j^{th}column in a 3   3 matrix and let $a_{ij} = -a_{ji}$ for every i and j then this matrix is an -

(A) orthogonal matrix
(B) singular matrix
(C) matrix whose principal diagonal elements are all zero
(D) skew symmetric matrix

25. Let A be an invertible matrix then which of the following is/are true :

(A) $|A^{-1}| = |A|^{-1}$
(B) $(A^2)^{-1} = (A^{-1})^2$
(C) $(A^T)^{-1} = (A^{-1})^T$
(D) none of these

26. If $A = \begin{bmatrix} 1 & 9 & -7 \\ i & \omega^n & 8 \\ 1 & 6 & \omega^{2n} \end{bmatrix}$, where $i = \sqrt{-1}$ and ω is complex cube root of unity, then tr(A) will be-

(A) 1, if $n = 3k,\ k \in N$
(B) 3, if $n = 3k,\ k \in N$
(C) 0, if $n \neq 3k,\ k \in N$
(D) –1, if $n \neq 3k,\ n \in N$

27. If A is a square matrix, then -

(A) AA^T is symmetric
(B) AA^T is skew-symmetric
(C) A^TA is symmetric
(D) A^TA is skew symmetric.

28. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ satisfies the equation $x^2 + k = 0$, then -

(A) $a + d = 0$
(B) $k = -|A|$
(C) $k = a^2 + b^2 + c^2 + d^2$
(D) $k = |A|$

29. If A and B are invertible matrices, which one of the following statement is/are correct -

(A) $Adj(A) = |A|A^{-1}$
(B) $\det(A^{-1}) = |\det(A)|^{-1}$
(C) $(A + B)^{-1} = B^{-1} + A^{-1}$
(D) $(AB)^{-1} = B^{-1}A^{-1}$

30. Matrix $\begin{bmatrix} a & b & (a\alpha - b) \\ b & c & (b\alpha - c) \\ 2 & 1 & 0 \end{bmatrix}$ is non invertible if -

(A) $\alpha = 1/2$
(B) a, b, c are in A.P.
(C) a, b, c are in G.P.
(D) a, b, c are in H.P.

CHECK YOUR GRASP — **ANSWER KEY** — EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A	A	A	C	C	B	B	D	A	A
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	C	D	D	A	B	D	B	B	D	D
Que.	21	22	23	24	25	26	27	28	29	30
Ans.	A	A,B	B,C	B,C,D	A,B,C	B,C	A,C	A,D	A,B,D	A,C

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. If A and B are square matrices of same order and $AA^T = I$ then $(A^TBA)^{10}$ is equal to -

(A) $AB^{10}A^T$ (B) $A^TB^{10}A$ (C) $A^{10}B^{10}(A^T)^{10}$ (D) $10A^TBA$

2. If A is a invertible idempotent matrix of order n, then adj A is equal to -

(A) $(\text{adj } A)^2$ (B) I (C) A^{-1} (D) none of these

3. Matrix $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if xyz = 60 and 8x + 4y + 3z = 20, then A (adj A) is equal to -

(A) $\begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$ (B) $\begin{bmatrix} 88 & 0 & 0 \\ 0 & 88 & 0 \\ 0 & 0 & 88 \end{bmatrix}$ (C) $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$ (D) $\begin{bmatrix} 34 & 0 & 0 \\ 0 & 34 & 0 \\ 0 & 0 & 34 \end{bmatrix}$

4. Let three matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$ then

$$t_r(A) + t_r\left(\frac{ABC}{2}\right) + t_r\left(\frac{A(BC)^2}{4}\right) + t_r\left(\frac{A(BC)^3}{8}\right) + \ldots\ldots + \infty =$$

(A) 6 (B) 9 (C) 12 (D) none of these

5. Let A, B, C, D be (not necessarily square) real matrices such that $A^T = BCD$; $B^T = CDA$; $C^T = DAB$ and $D^T = ABC$ for the matrix S = ABCD, then which of the following is/are true

(A) $S^3 = S$ (B) $S^2 = S^4$ (C) $S = S^2$ (D) none of these

6. If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ then let us define a function $f(x) = \det(A^TA^{-1})$ then which of the following can be the value of $\underbrace{f(f(f(f\ldots\ldots\ldots f(x))))}_{n \text{ times}}$ $(n \geq 2)$

(A) $f^n(x)$ (B) 1 (C) $f^{n-1}(x)$ (D) nf(x)

7. For a matrix $A = \begin{bmatrix} 1 & 2r-1 \\ 0 & 1 \end{bmatrix}$, the value of $\prod_{r=1}^{50}\begin{bmatrix} 1 & 2r-1 \\ 0 & 1 \end{bmatrix}$ is equal to -

(A) $\begin{bmatrix} 1 & 100 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 4950 \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 5050 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 2500 \\ 0 & 1 \end{bmatrix}$

8. If A and B are two invertible matrices of the same order, then adj (AB) is equal to -

(A) adj (B) adj (A) (B) $|B||A|\ B^{-1}\ A^{-1}$ (C) $|B||A|\ A^{-1}\ B^{-1}$ (D) $|A||B|(AB)^{-1}$

9. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$, then -

(A) a = 1, c = –1 (B) a = 2, $c = -\frac{1}{2}$ (C) a = –1, c = 1 (D) $a = \frac{1}{2}$, $c = \frac{1}{2}$

10. If A and B are different matrices satisfying $A^3 = B^3$ and $A^2B = B^2A$, then which of the following is/are incorrect-

(A) det $(A^2 + B^2)$ must be zero

(B) det (A – B) must be zero

(C) det $(A^2 + B^2)$ as well as det (A – B) must be zero

(D) At least one of det $(A^2 + B^2)$ or det (A – B) must be zero

11. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then-

(A) AdjA is zero matrix

(B) $\text{Adj}\,A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

(C) $A^{-1} = A$

(D) $A^2 = I$

12. If A and B are square matrices of the same order such that $A^2 = A$, $B^2 = B$, $AB = BA$, then which one of the following may be true-

(A) $A(B)^2 = O$ (B) $(A + B)^2 = A + B$ (C) $(A - B)^2 = A - B$ (D) none of these

13. If B is an idempotent matrix and $A = I - B$, then-

(A) $A^2 = A$ (B) $A^2 = I$ (C) $AB = O$ (D) $BA = O$

14. Let $\Delta_0 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ (where $\Delta_0 \neq 0$) and let Δ_1 denote the determinant formed by the cofactors of elements of Δ_0 and Δ_2 denote the determinant formed by the cofactor at Δ_1 and so on Δ_n denotes the determinant formed by the cofactors at Δ_{n-1} then the determinant value of Δ_n is -

(A) Δ_0^{2n} (B) $\Delta_0^{2^n}$ (C) $\Delta_0^{n^2}$ (D) Δ_0^2

BRAIN TEASERS				ANSWER KEY					EXERCISE-2
Que.	1	2	3	4	5	6	7	8	9	10
Ans.	B	A,B,C	C	A	A,B	A,B,C	D	A,B,D	A	A,B,C
Que.	11	12	13	14						
Ans.	B,C,D	A,B,C	A,C,D	B						

EXERCISE - 03

MISCELLANEOUS TYPE QUESTIONS

TRUE / FALSE

1. Let A, B be two matrices such that they commute, then $A^2 - B^2 = (A - B)(A + B)$
2. If A is a periodic matrix with period 2 then $A^6 = A$.
3. Let A, B be two matrices such that they commute, then $(AB)^n = A^nB^n$.
4. All positive odd integral powers of skew symmetric matrix are symmetric.
5. Let A, B be two matrices, such that AB = A and BA = B, then $A^2 = A$ and $B^2 = B$.
6. If A & B are symmetric matrices of same order then AB – BA is symmetric.
7. If A and B are square matrices of order n, then A and B will commute, iff $A - \lambda I$ and $B - \lambda I$ commute for every scalar λ.

MATCH THE COLUMN

Following questions contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1.

	Column-I		Column-II
	Matrix		Type of matrix
(A)	$\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$	(p)	Idempotent
(B)	$\begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$	(q)	Involutary
(C)	$\frac{1}{3}\begin{bmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ -2 & -2 & -1 \end{bmatrix}$	(r)	Nilpotent
(D)	$\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$	(s)	Orthogonal

2.

	Column-I		Column-II
(A)	If A is a square matrix of order 3 and det A = 162 then $\det\left(\frac{A}{3}\right) =$	(p)	6
(B)	If A is a matrix such that $A^2 = A$ and $(I + A)^5 = I + \lambda A$ then $\frac{2\lambda + 1}{7}$	(q)	5
(C)	If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$ and $A^2 - xA + yI = 0$ then $y - x =$	(r)	0
(D)	If $A = \begin{bmatrix} 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \\ 17 & 18 & 19 & 20 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 & 5 & 7 \\ -3 & -3 & -10 & -10 \\ 5 & 10 & 5 & 0 \\ 7 & 10 & 0 & 7 \end{bmatrix}$ then $(AB)_{23}$	(s)	9

ASSERTION & REASON

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for statement-I

(C) Statement-I is true, Statement-II is false

(D) Statement-I is false, Statement-II is true

1. **Statement - I** : If a, b, c are distinct real number and x, y, z are not all zero given that $ax + by + cz = 0$, $bx + cy + az = 0$, $cx + ay + bz = 0$, then $a + b + c \neq 0$

Statement - II : If a, b, c are distinct positive real number then $a^2 + b^2 + c^2 \neq ab + bc + ca$.

(A) A (B) B (C) C (D) D

2. **Statement - I** : If A is skew symmetric matrix of order 3 then its determinant should be zero

Statement - II : If A is square matrix, then det A = det A' = det(–A')

(A) A (B) B (C) C (D) D

3. **Statement-I** : If A is a non-singular symmetric matrix, then its inverse is also symmetric.

Because

Statement-II : $(A^{-1})^T = (A^T)^{-1}$, where A is a non-singular symmetric matrix.

(A) A (B) B (C) C (D) D

4. **Statement - I** : There are only finitely many 2 2 matrices which commute with the matrix $\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$.

Because

Statement - II : If A is non-singular, then it commutes with I, adj A and A^{-1}.

(A) A (B) B (C) C (D) D

5. **Statement-I** : If $x = \begin{bmatrix} \frac{\sqrt{3}+1}{2\sqrt{2}} & \frac{\sqrt{3}-1}{2\sqrt{2}} \\ \frac{1-\sqrt{3}}{2\sqrt{2}} & \frac{\sqrt{3}+1}{2\sqrt{2}} \end{bmatrix} A \begin{bmatrix} \frac{\sqrt{3}+1}{2\sqrt{2}} & \frac{1-\sqrt{3}}{2\sqrt{2}} \\ \frac{\sqrt{3}-1}{2\sqrt{2}} & \frac{\sqrt{3}+1}{2\sqrt{2}} \end{bmatrix}$ & if A is idempotent matrix then x is also idempotent matrix.

Because

Statement-II : If P is an orthogonal matrix & $Q = PAP^T$, then $Q^n = PA^nP^T$.

(A) A (B) B (C) C (D) D

6. **Statement-I** : The determinants of a matrix $A = [a_{ij}]_{5 \quad 5}$ where $a_{ij} + a_{ji} = 0$ for each i and j is zero.

Because

Statement-II : The determinant of a skew symmetric matrix of odd order is zero.

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1

Let P(x, y) be any point and P'(x_1, y_1) be its image in x-axis then

$x_1 = x$

$y_1 = -y$

This system of equation is equivalent to the matrix equation.

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = A\begin{bmatrix} x \\ y \end{bmatrix}$$

where A is a square matrix of order 2

Similarly $\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = B\begin{bmatrix} x \\ y \end{bmatrix}$, $\begin{bmatrix} x_3 \\ y_3 \end{bmatrix} = C\begin{bmatrix} x \\ y \end{bmatrix}$, $\begin{bmatrix} x_4 \\ y_4 \end{bmatrix} = D\begin{bmatrix} x \\ y \end{bmatrix}$

represents the reflection of point (x, y) in y-axis, origin and the line y = x respectively.

On the basis of above information, answer the following questions :

1. The value of A + B + C + D is -

(A) $\begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

2. Let X be a square matrix given by X = A + AD^2 + AD^4 +.........+AD^{2n-2}, then X is -

(A) $\begin{bmatrix} -n & 0 \\ 0 & -n \end{bmatrix}$ (B) $\begin{bmatrix} n & 0 \\ 0 & n \end{bmatrix}$ (C) $\begin{bmatrix} n & 0 \\ 0 & -n \end{bmatrix}$ (D) $\begin{bmatrix} -n & 0 \\ 0 & n \end{bmatrix}$

3. Let P(a, b) be a point & $\begin{bmatrix} x \\ y \end{bmatrix} = DCBA\begin{bmatrix} a \\ b \end{bmatrix}$ then Q(x, y) represents the reflection of point P(a, b) in -

(A) x-axis (B) y-axis (C) origin (D) line y = x

Comprehension # 2

Matrix A is called orthogonal matrix if $AA^T = I = A^TA$. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ be an orthogonal matrix. Let $\vec{a} = a_1\tilde{i} + a_2\tilde{j} + a_3\tilde{k}$, $\vec{b} = b_1\tilde{i} + b_2\tilde{j} + b_3\tilde{k}$, $\vec{c} = c_1\tilde{i} + c_2\tilde{j} + c_3\tilde{k}$. Then $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$ & $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$ i.e. $\vec{a}, \vec{b}$ & $\vec{c}$ forms mutually perpendicular triad of unit vectors.

If abc = p and $Q = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}$, where Q is an orthogonal matrix. Then.

On the basis of above information, answer the following questions :

1. The values of a + b + c is -

(A) 2 (B) p (C) 2p (D) ±1

2. The values of ab + bc + ca is -

(A) 0 (B) p (C) 2p (D) 3p

3. The value of $a^3 + b^3 + c^3$ is -

(A) p (B) 2p (C) 3p (D) None of these

4. The equation whose roots are a, b, c is -

(A) $x^3 - 2x^2 + p = 0$ (B) $x^3 - px^2 + px + p = 0$

(C) $x^3 - 2x^2 + 2px + p = 0$ (D) $x^3 \pm x^2 - p = 0$

Comprehension # 3

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = \text{adj}(B_{n-1})$, $n \in N$ and I is an identity matrix of order 3.

On the basis of above information, answer the following questions :

1. $\det.(A_0 + A_0^2B_0^2 + A_0^3 + A_0^4B_0^4 + ...10$ terms) is equal to -

(A) 1000 (B) –800 (C) 0 (D) –8000

2. $B_1 + B_2 + ... + B_{49}$ is equal to -

(A) B_0 (B) $7B_0$ (C) $49B_0$ (D) 49I

3. For a variable matrix X the equation $A_0X = B_0$ will have -

(A) unique solution (B) infinite solution (C) finitely many solution (D) no solution

MISCELLANEOUS TYPE QUESTION — **ANSWER KEY** — EXERCISE-3

- ***True / False***

  1. T 2. F 3. T 4. F 5. T 6. F 7. T

- ***Match the Column***

  1. (A) → (p), (B) → (q), (C) → (s), (D) → (r) 2. (A) → (p), (B) → (s), (C) → (q), (D) → (r)

- ***Assertion & Reason***

  1. D 2. C 3. A 4. D 5. A 6. A

- ***Comprehension Based Questions***

  Comprehension # 1 : 1. B 2. C 3. D

  Comprehension # 2 : 1. D 2. A 3. D 4. D

  Comprehension # 3 : 1. C 2. C 3. D

EXERCISE - 04 [A]

CONCEPTUAL SUBJECTIVE EXERCISE

1. Find the value of x and y that satisfy the equations -

$$\begin{bmatrix} 3 & -2 \\ 3 & 0 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} y & y \\ x & x \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3y & 3y \\ 10 & 10 \end{bmatrix}$$

2. A is a square matrix of order n.

ℓ = maximum number of distinct entries if A is a triangular matrix

m = maximum number of distinct entries if A is a diagonal matrix

p = minimum number of zeroes if A is a triangular matrix

If $\ell + 5 = p + 2m$, find the order of the matrix.

3. If the matrices $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(a, b, c, d not all simultaneously zero) commute, find the value of $\frac{d-b}{a+c-b}$. Also show that the matrix which commutes with A is of the form $\begin{bmatrix} \alpha-\beta & 2\beta/3 \\ \beta & \alpha \end{bmatrix}$

4. Consider the two matrices A and B where $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$; $B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$. Let n(A) denotes the number of elements in A. When the two matrices X and Y are not conformable for multiplication then n(XY) = 0

If C = (AB)(B'A); D = (B'A)(AB) then, find the value of $\left(\frac{n(C)\left((|D|^2 + n(D)\right)}{n(A) - n(B)}\right)$.

5. Define $A = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$. Find a vertical vector V such that $(A^8 + A^6 + A^4 + A^2 + I)V = \begin{bmatrix} 0 \\ 11 \end{bmatrix}$

(where I is the 2 2 identity matrix).

6. If A is an idempotent matrix and I is an identity matrix of the same order, find the value of n, $n \in N$, such that $(A + I)^n = I + 127A$.

7. If the matrix A is involutary, show that $\frac{1}{2}(I + A)$ and $\frac{1}{2}(I - A)$ are idempotent and $\frac{1}{2}(I + A) \cdot \frac{1}{2}(I - A) = 0$.

8. Let X be the solution set of the equation $A^x = I$, where $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$ and I is the corresponding unit matrix and $x \in N$ then find the minimum value of $\sum(\cos^x\theta + \sin^x\theta), \theta \in R$.

9. Express the matrix $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & -6 \\ -1 & 0 & 4 \end{bmatrix}$ as a sum of a lower triangular matrix & an upper triangular matrix with zero in leading diagonal of upper triangular matrix. Also express the matrix as a sum of a symmetric and a skew symmetric matrix.

10. Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 2 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$. Find P such that $BPA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

11. If $A = \begin{bmatrix} \sin\alpha & -\cos\alpha & 0 \\ \cos\alpha & \sin\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then find $|A^T|$ and $|A^{-1}|$.

12. Show that, $\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

13. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then show that $F(x).F(y) = F(x + y)$. Hence prove that $[F(x)]^{-1} = F(-x)$.

14. If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, then show that the matrix A is a root of the polynomial $f(x) = x^3 - 6x^2 + 7x + 2$.

15. Use matrix to solve the following system of equations

(a) $x + y + z = 3$, $x + 2y + 3z = 4$, $x + 4y + 9z = 6$

(b) $x + y + z = 6$, $x - y + z = 2$, $2x + y - z = 1$

(c) $x + y + z = 3$, $x + 2y + 3z = 4$, $2x + 3y + 4z = 7$

(d) $x + y + z = 3$, $x + 2y + 3z = 4$, $2x + 3y + 4z = 9$

CONCEPTUAL SUBJECTIVE EXERCISE — ANSWER KEY — EXERCISE-4(A)

1. $x = 3/2,\ y = 2$ **2.** 4 **3.** 1 **4.** 650 **5.** $V = \begin{bmatrix} 0 \\ 1 \\ \overline{11} \end{bmatrix}$ **6.** $n = 7$

8. 2 **9.** $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ -1 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 5 \\ 0 & 0 & -6 \\ 0 & 0 & 0 \end{bmatrix}$; $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & -3 \\ 2 & -3 & 4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 3 \\ 0 & 0 & -3 \\ -3 & 3 & 0 \end{bmatrix}$ **10.** $\begin{bmatrix} -4 & 7 & -7 \\ 3 & -5 & 5 \end{bmatrix}$ **11.** 1, 1

15. (a) $x = 2,\ y = 1,\ z = 0$ (b) $x = 1,\ y = 2,\ z = 3$

(c) $x = 2 + k,\ y = 1 - 2k,\ z = k$ where $k \in R$ (d) inconsistent, hence no solution

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then prove that value of f and g satisfying the matrix equation $A^2 + fA + gI = O$ are equal to $-t_r(A)$ and determinant of A respectively. Given a, b, c, d are non zero reals and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

2. $A_{3\ \ 3}$ is a matrix such that $|A|=a$, $B = (\text{adj } A)$ such that $|B| = b$. Find the value of $(ab^2 + a^2b + 1)S$ where $\frac{1}{2}S = \frac{a}{b} + \frac{a^2}{b^3} + \frac{a^3}{b^5} + \ldots\ldots.$ up to ∞, and $a = 3$.

3. Find the number of 2 2 matrix satisfying :

   (a) a_{ij} is 1 or -1 ; (b) $a_{11}^2 + a_{12}^2 = a_{21}^2 + a_{22}^2 = 2$; (c) $a_{11}a_{21} + a_{12}a_{22} = 0$

4. If A is a skew symmetric matrix and I + A is non singular, then prove that the matrix $B = (I - A)(I + A)^{-1}$ is an orthogonal matrix. Use this to find a matrix B given $A = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$.

5. Given $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 9 & 3 \\ 3 & 1 \end{bmatrix}$. I is a unit matrix of order 2. Find all possible matrix X in the following cases.

   (a) $AX = A$ (b) $XA = I$(c) $XB = O$ but $BX \neq O$.

6. Determine the values of a and b for which the system $\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$

   (a) has a unique solution ; (b) has no solution and (c) has infinitely many solutions

7. If A is an orthogonal matrix and B = AP where P is a non singular matrix then show that the matrix PB^{-1} is also orthogonal.

8. If $\begin{bmatrix} 1 & 2 & a \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 18 & 2007 \\ 0 & 1 & 36 \\ 0 & 0 & 1 \end{bmatrix}$ then find a + n.

9. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $P = \begin{bmatrix} p \\ q \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Such that AP = P and a + d = 5050. Find the value of (ad − bc).

BRAIN STORMING SUBJECTIVE EXERCISE — ANSWER KEY — EXERCISE-4(B)

2. 225 3. 8 4. $\frac{1}{13}\begin{bmatrix} -12 & -5 \\ 5 & -12 \end{bmatrix}$

5. (a) $X = \begin{bmatrix} a & b \\ 2-2a & 1-2b \end{bmatrix}$ for $a, b \in R$; (b) X does not exist ; (c) $X = \begin{bmatrix} a & -3a \\ c & -3c \end{bmatrix}$ $a, c \in R$ and $3a + c \neq 0$

6. (a) $a \neq -3$; $b \in R$; (b) $a = -3$ and $b \neq 1/3$; (c) $a = -3$, $b = 1/3$ 8. 200 9. 5049

EXERCISE - 05 [A]

JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ then [AIEEE 2003]

(1) $\alpha = a^2 + b^2, \beta = a^2 - b^2$
(2) $\alpha = a^2 + b^2, \beta = ab$
(3) $\alpha = a^2 + b^2, \beta = 2ab$
(4) $\alpha = 2ab, \beta = a^2 + b^2$

2. If $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ then- [AIEEE 2004]

(1) A^{-1} does not exist
(2) $A^2 = I$
(3) $A = 0$
(4) $A = (-1)\ I$

3. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ where $B = A^{-1}$, then α is equal to- [AIEEE 2004]

(1) 2
(2) –1
(3) –2
(4) 5

4. If $A^2 - A + I = 0$, then the inverse of A [AIEEE 2005]

(1) I – A
(2) A – I
(3) A
(4) A + I

5. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, (by the principal of mathematical induction) [AIEEE-2005]

(1) $A^n = nA - (n-1)I$
(2) $A^n = 2^{n-1}A + (n-1)I$
(3) $A^n = nA + (n-1)I$
(4) $A^n = 2^{n-1}A - (n-1)I$

6. If A and B are square matrices of size n   n such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true- [AIEEE- 2006]

(1) AB = BA
(2) Either of A or B is a zero matrix
(3) Either of A or B is an identity matrix
(4) A = B

7. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in N$. Then- [AIEEE- 2006]

(1) there exist more than one but finite number of B's such that AB = BA
(2) there exist exactly one B such that AB = BA
(3) there exist infinitely many B's such that AB=BA
(4) there cannot exist any B such that AB = BA

8. Let $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ If $|A^2| = 25$, then $|\alpha|$ equals- [AIEEE- 2006]

(1) 5^2
(2) 1
(3) 1/5
(4) 5

9. Let A be a 2   2 matrix with real entries. Let I be the 2   2 identity matrix. Denoted by tr(A), the sum of diagonal entries of A. Assume that $A^2 = I$.

Statement –1: If $A \neq I$ and $A \neq -I$, then det A = –1 [AIEEE- 2008]

Statement –2 : If $A \neq I$ and $A \neq -I$, then tr(A) $\neq 0$.

(1) Statement –1 is false, Statement –2 is true.
(2) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.
(3) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1.
(4) Statement–1 is true, Statement–2 is false

10. Let A be a square matrix all of whose entries are integers. Then which one of the following is true? [AIEEE- 2008]

(1) If det A = ± 1, then A^{-1} exists but all its entries are not necessarily integers

(2) If det A ≠ ±1, then A^{-1} exists and all its entries are non–integers

(3) If det A = ±1, then A^{-1} exists and all its entries are integers

(4) If det A = ±1, then A^{-1} need not exist

11. Let A be a 2 2 matrix [AIEEE- 2009]

Statement–1 : adj (adj A) = A

Statement–2 : | adj A | = |A|

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for statement–1.

12. The number of 3 3 non-singular matrices, with four entries as 1 and all other entries as 0, is :- [AIEEE-2010]

(1) Less than 4 (2) 5 (3) 6 (4) At least 7

13. Let A be a 2 2 matrix with non-zero entries and let A^2 = I, where I is 2 2 identity matrix. Define Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A. [AIEEE-2010]

Statement–1 : Tr(A) = 0.

Statement–2 : |A| = 1.

(1) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(2) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for statement–1.

(3) Statement–1 is true, Statement–2 is false.

(4) Statement–1 is false, Statement–2 is true.

14. Let A and B be two symmetric matrices of order 3.

Statement-1 : A(BA) and (AB)A are symmetric matrices.

Statement-2 : AB is symmetric matrix if matrix multiplication of A with B is commutative. [AIEEE-2011]

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

15. **Statement-1** : Determinant of a skew-symmetric matrix of order 3 is zero.

Statement-1 : For any matrix A, det(A^T) = det(A) and det(–A) = –det(A).

Where det(B) denotes the determinant of matrix B. Then : [AIEEE-2011]

(1) Statement-1 is true and statement-2 is false (2) Both statements are true

(3) Both statements are false (4) Statement-1 is false and statement-2 is true.

16. If ω ≠ 1 is the complex cube root of unity and matrix $H=\begin{bmatrix}\omega & 0\\ 0 & \omega\end{bmatrix}$, then H^{70} is equal to: [AIEEE-2011]

(1) H (2) 0 (3) –H (4) H^2

17. Let P and Q be 3 3 matrices with P ≠ Q. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to : [AIEEE-2012]

(1) –1 (2) –2 (3) 1 (4) 0

18. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to : **[AIEEE-2012]**

(1) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (3) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (4) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

PREVIOUS YEARS QUESTIONS **ANSWER KEY** EXERCISE-5 [A]

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	3	2	4	1	1	1	3	3	4	3
Que.	11	12	13	14	15	16	17	18		
Ans.	4	4	3	4	1	1	4	1		

EXERCISE - 05 [B] JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

1. If matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a,b,c are real positive numbers, abc = 1 and $A^T A = I$, then find the value of $a^3 + b^3 + c^3$. [JEE 2003, Mains 2M out of 60]

2. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then α is equal to - [JEE 2004 (Screening)]

(A) ±3 (B) ±2 (C) ±5 (D) 0

3. If M is a 3 3 matrix, where $M^TM = I$ and det (M) = 1, then prove that det (M–I) = 0.
[JEE 2004 (Mains), 2M out of 60]

4. $A = \begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}$, $B = \begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix}$, $U = \begin{bmatrix} f \\ g \\ h \end{bmatrix}$, $V = \begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

If AX = U has infinitely many solutions, then prove that BX = V cannot have a unique solution. If further $afd \neq 0$, then prove that BX = V has no solution [JEE 2004 (Mains), 4M out of 60]

5. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{6}(A^2 + cA + dI)$, then the value of c and d are -

[JEE 2005 (Screening)]

(A) –6, –11 (B) 6, 11 (C) –6, 11 (D) 6, –11

6. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $x = P^T Q^{2005} P$, then x is equal to -

[JEE 2005 (Screening)]

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$

(C) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$ (D) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

Comprehension (3 questions)

7. $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$, if U_1, U_2 and U_3 are columns matrices satisfying. $AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and U is 3 3 matrix whose columns are U_1, U_2, U_3 then answer the following questions -

(a) The value of |U| is -

(A) 3 (B) –3 (C) 3/2 (D) 2

(b) The sum of the elements of U^{-1} is -

(A) –1 (B) 0 (C) 1 (D) 3

(c) The value of $\begin{bmatrix}3 & 2 & 0\end{bmatrix} U \begin{bmatrix}3\\2\\0\end{bmatrix}$ is -

(A) [5] (B) [5/2] (C) [4] (D) [3/2]

[JEE 2006, 5 marks each]

8. Match the Statement / Expressions in Column I with the Statements / Expressions in Column II and indicate your answer by darkening the appropriate bubbles in the 4 4 matrix given in the ORS.

Column I	Column II
(A) The minimum value of $\frac{x^2+2x+4}{x+2}$ is	(P) 0
(B) Let A and B be 3 3 matrices of real numbers, where A is symmetric, B is skew-symmetric, and (A+B)(A–B) = (A – B) (A + B). If $(AB)^t = (-1)^k AB$, where $(AB)^t$ is the transpose of the matrix AB, then the possible values of k are	(Q) 1
(C) Let a = $\log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3^{-a})} < 2$, must be less than	(R) 2
(D) If $\sin\theta = \cos\phi$, then the possible values of $\frac{1}{\pi}\left(\theta-\phi-\frac{\pi}{2}\right)$ are	(S) 3

[JEE 2008, 6]

9. Let $\mathcal{A}$ be the set of all 3 3 symmetric matrices all of whose entries are either 0 or 1. Five of these entries are 1 and four of them are 0.

(a) The number of matrices in $\mathcal{A}$ is -

(A) 12 (B) 6 (C) 9 (D) 3

(b) The number of matrices A in $\mathcal{A}$ for which the system of linear equations

$$A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

has a unique solution, is -

(A) less than 4 (B) at least 4 but less than 7

(C) at least 7 but less than 10 (D) at least 10

(c) The number of matrices A in $\mathcal{A}$ for which the system of linear equations

$$A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

is inconsistent, is -

(A) 0 (B) more than 2 (C) 2 (D) 1

[JEE 2009, 4+4+4]

10. (a) The number of 3 × 3 matrices A whose entries are either 0 or 1 and for which the system $A\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ has exactly two distinct solutions, is

(A) 0 (B) $2^9 - 1$ (C) 168 (D) 2

(b) Let k be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}.$$

If det (adj A) + det(adj B) = 10^6, then [k] is equal to

[**Note** : adj M denotes the adjoint of a square matrix M and [k] denotes the largest integer less than or equal to k].

(c) Let p be an odd prime number and T_p be the following set of 2 × 2 matrices :

$$T_p = \left\{ A = \begin{bmatrix} a & b \\ c & a \end{bmatrix} : a, b, c \in \{0, 1, 2, \ldots\ldots, p-1\} \right\}$$

(i) The number of A in T_p such that A is either symmetric or skew-symmetric or both, and det(A) divisible by p is -

(A) $(p - 1)^2$ (B) $2(p - 1)$ (C) $(p - 1)^2 + 1$ (D) $2p - 1$

(ii) The number of A in T_p such that the trace of A is not divisible by p but det (A) is divisible by p is -

[**Note** : The trace of a matrix is the sum of its diagonal entries.]

(A) $(p - 1)(p^2 - p + 1)$ (B) $p^3 - (p - 1)^2$

(C) $(p - 1)^2$ (D) $(p - 1)(p^2 - 2)$

(iii) The number of A in T_p such that det (A) is not divisible by p is -

(A) $2p^2$ (B) $p^3 - 5p$ (C) $p^3 - 3p$ (D) $p^3 - p^2$

[JEE 2010, 3+3+3+3+3]

11. Let M and N be two 3 3 non-singular skew-symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to - [JEE 2011, 4]

(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

12. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form $\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a,b and c is either ω or ω^2. Then the number of distinct matrices in the set S is-

(A) 2 (B) 6 (C) 4 (D) 8

[JEE 2011, 3, (–1)]

13. Let M be 3 3 matrix satisfying

$$M\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}, M\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \text{ and } M\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \end{bmatrix}$$

Then the sum of the diagonal entries of M is [JEE 2011, 4]

14. Let P =$[a_{ij}]$ be a 3 3 matrix and let Q = $[b_{ij}]$, where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \leq i, j \leq 3$. If the determinant of P is 2, then the determinant of the matrix Q is - [JEE 2012, 3M, –1M]

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

15. If P is a 3 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that [JEE 2012, 3M, –1M]

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) PX = X (C) PX = 2X (D) PX = –X

16. If the adjoint of a 3 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 2 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are) - [JEE 2012, 4M]

(A) –2 (B) –1 (C) 1 (D) 2

PREVIOUS YEARS QUESTIONS — ANSWER KEY — EXERCISE-5 [B]

1.	4	2.	A	5.	C	6.	A	7.	(a) A, (b) B, (c) A
8.	(A) R (B) Q,S (C) R,S (D) P,R					9.	(a) A, (b) B, (c) B		
10.	(a) A, (b) 4; (c) (i) D, (ii) C, (iii) D					11.	Bonus	12.	A
13.	9								
14.	D	15.	D	16.	A,D				